1. Introduction

The electrification of automotive powertrain systems is a focus of increasing attention as a means of helping to resolve the issues of global warming, because electric automotive powertrain systems have a lower environmental load compared to internal combustion engines. In addition, independent installation of electric motors enables more flexible vehicle packaging and driving force distribution control. The electric motor installed inside each wheel is called an in-wheel motor, and a vehicle equipped with in-wheel motors is expected as a future electric vehicle.

In the past, various driving force distribution controls have been studied in terms of vehicle maneuverability and safety. However, since the improvement of the miles per charge is necessary in order to broaden the use of electric vehicles, the additional power required to achieve vehicle dynamics control and the possibility of optimizing the cornering efficiency should be clarified. Although some reports have developed control methods to reduce energy consumption while turning, the energy loss mechanism due to turning has not yet been formulated. Therefore, in order to clarify the cornering loss and realize efficient direct yaw moment control (DYC), as a first step, the present paper describes the formulation of power consumption in steady-state cornering. Validation tests are conducted using an actual vehicle equipped with in-wheel motors.

2. Simulation Model

2.1 Driving Resistance while Turning

In the present paper, a linear two-wheel model is used to formulate the driving resistance while turning, as shown in Fig. 1. The equations of motion are expressed as follows:

Longitudinal:
\[ m(\ddot{u} - vr) = F_x - F_{x'} - 2F_y' \sin \delta_f + 2F_y' \sin \delta_r, \]

Lateral:
\[ m(\ddot{v} + ur) = 2F_y' \cos \delta_f + 2F_y' \cos \delta_r, \]

Yaw:
\[ I_\gamma \dot{\gamma} = 2l_x F_{y'} \cos \delta_f - 2l_x F_{y'} \cos \delta_r + M_z. \]

In the above equations, \(m\) is the vehicle mass, \(I_z\) is the yaw inertia, \(l_x\) and \(l\) are the distances from the gravity center to the front and rear axles, respectively, \(\delta_f\) and \(\delta_r\) are the front and rear actual steer angles, respectively, \(u\) and \(v\) are the longitudinal and lateral velocities, respectively, \(r\) is the yaw rate, \(F_x\) is the total driving force, \(F_{y'}\) and \(F_{y'}\) are the front and rear lateral forces,
respectively, and \( M_z \) is the direct yaw moment. The rolling resistance \( F_{rr} \) and aerodynamic resistance \( F_{ar} \) are expressed as follows:

\[
F_{rr} = \mu_r m g, \quad F_{ar} = \rho A D V^2 / 2, \tag{4}
\]

where \( \mu_r \) is the rolling resistance coefficient, \( g \) is gravitational acceleration, \( \rho \) is the air density, \( A \) is the frontal projected area, \( C_D \) is the aerodynamic resistance coefficient, and \( V \) is the vehicle velocity, which is equal to the longitudinal velocity \( u \).

In steady-state cornering and linear conditions, the total driving force can be obtained by substituting the following equations into the equations of motion:

\[
\dot{u} = 0, \quad \dot{v} = 0, \quad \dot{r} = 0, \\
u = V \cos \beta \equiv V, \quad v = V \sin \beta \equiv V \beta, \\
\cos \delta_f \approx 1, \quad \cos \delta_r \approx 1, \\
\sin \delta_f \approx \delta_f, \quad \sin \delta_r \approx \delta_r, \tag{5}
\]

where \( \beta \) is the body slip angle.

The total driving force is given by

\[
F_z = F_{rr} + F_{ar} + 2F_{rr} \left( \delta_f - \beta \right) + 2F_{ar} \left( \delta_r - \beta \right). \tag{6}
\]

In the present paper, we define the cornering resistance as follows:

\[
F_{cr} = 2F_{rr} \left( \delta_f - \beta \right) + 2F_{ar} \left( \delta_r - \beta \right). \tag{7}
\]

Equation (7) indicates that the cornering resistance interferes with the vehicle longitudinal motion due to the front and rear lateral forces.

Then, the respective front and rear steer angles and body slip angle can be expressed as follows, using the linear two-wheel model:

\[
\delta_f - \delta_r = 1 - \frac{m l_1 K_r - l_1 K_f}{2l^2 - l_1^2} \frac{l}{R} \frac{K_r + K_f}{2l^2 K_f K_r}, \tag{8}
\]

\[
\beta = 1 - \frac{m l_2 K_f - l_2 K_r}{2l^2} \frac{V^2}{K_f K_r} \\
\left[ \frac{1 - m l_2 V^2}{2l_2 l_1 K_f} \frac{l}{l_1} \delta_f + \frac{1 - m l_1 V^2}{2l_1 l_2 K_f} \frac{l}{l_2} \delta_r \\
- \frac{m V^2 + 2 l_1 K_r - l_2 K_f}{4l^2 K_f K_r} M_z \right]. \tag{9}
\]

where \( \delta_f \) and \( \delta_r \) are the front and rear cornering stiffnesses of the tires, respectively, and \( l \) is the wheelbase. Using Eqs. (8) and (9), the cornering resistance can be obtained again as follows:

\[
F_{cr} = \frac{l_1^2 + l_2^2}{\left( \frac{1}{K_f} + \frac{1}{K_r} \right) 2l^2} \left( \frac{m A}{l} \right)^2 V \left[ \frac{1}{R} \left( \frac{l_2}{K_f} - \frac{l_1}{K_r} \right) \frac{m A}{l_1} M_z \right] + \left( \frac{1}{K_f} + \frac{1}{K_r} \right) M_z^2. \tag{10}
\]

The first term in Eq. (10) indicates that the cornering resistance increases with lateral acceleration and that higher cornering stiffness is desirable. The second term indicates that the direct yaw moment can reduce the cornering resistance greatly if the cornering radius is small and the difference between the front and rear tire cornering stiffnesses is large. The third term indicates that a higher cornering stiffness can suppress the increase in the cornering resistance caused by the direct yaw moment.

2.2 Power Consumption while Turning

The total mechanical power \( P_v \) can be expressed as follows:

\[
P_v = \sum_{j=1}^{4} \tau_j \omega_j. \tag{11}
\]

Here, the driving torque and angular velocity of each wheel are represented as \( \tau_j \) and \( \omega_j \), respectively ( \( j = 1 \): front left, 2: front right, 3: rear left, 4: rear
By substituting Eq. (10) into Eq. (16), the power of the cornering resistance can be obtained as follows:

\[
P_{cr} = \left( F_{cr} + \frac{M}{R} \right) V = \left( \frac{l_f}{K_f} + \frac{l_r}{K_r} \right) \left( mA \right)^2 - \left( \frac{l_f}{K_f} - \frac{l_r}{K_r} \right) \frac{mA}{l^2} M_z \left( \frac{1}{K_f} + \frac{1}{K_r} \right) \frac{M^2}{l^2} \right) V, \quad (17)
\]

As indicated by the second term in Eq. (17), unlike the cornering resistance, the reduction effect of the cornering resistance cancels out the additional power caused by the direct yaw moment, with the result that the reduction effect of the power depends only on the difference between the front and rear tire cornering stiffnesses. The other terms are the same as for the cornering resistance. If a large yaw moment is added, the longitudinal tire slip power increases and the difference between the inner and outer longitudinal slips increases.

### 2.3 Mechanical Resistance of the Reduction Gear

In the in-wheel motor unit of the test vehicle, the motor torque is transmitted to the wheel through a counter gear and planetary gear. Therefore, by adding the mechanical resistance of the reduction gear to the driving resistance, the motor torque \( \tau_{mj} \) can be expressed as follows:

\[
\tau_{mj} = \begin{cases} 
F_{xj} + s_{xj} \frac{V_j}{r_j} & (j = 1, 3) \\
F_{xo} + s_{xo} \frac{V_o}{r_o} & (j = 2, 4) 
\end{cases} 
\]

Thus, the angular velocity can be expressed in terms of the longitudinal tire slip as follows:

\[
\omega_j = \begin{cases} 
\left( 1 + s_j \right) \frac{V_j}{r_j} & (j = 1, 3) \\
\left( 1 + s_j \right) \frac{V_o}{r_o} & (j = 2, 4) 
\end{cases} 
\]

where \( s_i \) and \( s_o \) are the longitudinal tire slips at the inner and outer wheels, respectively. By substituting Eqs. (12) through (15) into Eq. (11), the total mechanical power can be expressed as follows:

\[
P_{\text{mech}} = P_{rr} + P_{aw} + P_{ar} + P_{sy}, \quad (16)
\]

In the above equation, the power of the rolling resistance, aerodynamic resistance, cornering resistance, and longitudinal tire slip are denoted as \( P_{rr}, P_{aw}, P_{ar}, \) and \( P_{sy}, \) respectively.
Then, the mechanical loss of the reduction gear can be obtained as follows:

\[ L_{mg} = \tau_r \omega \frac{\left( \eta_s^{-1} - 1 \right) \tau_r \omega}{\left( \eta_s^{-1} - 1 \right) \tau_r \omega} \left( \tau_r > 0 \right) \left( \eta_s^{-1} - 1 \right) \tau_r \omega \left( \tau_r < 0 \right). \]  

(21)

Figure 2 shows the change in mechanical resistance caused by the driving force distribution. Although the mechanical resistance remains low at steady-state driving force \( F_x^0 \), the mechanical resistance per wheel increases, provided that the inner and outer driving forces work in opposite directions. The mechanical loss has the same characteristics as the mechanical resistance.

2.4 Electric Loss of the Motor and Inverter

An interior permanent magnet synchronous motor (IPMSM) model was constructed based on equivalent circuit theory, as shown in Fig. 3. According to this theory, the equivalent circuit equations can be expressed as shown in the following equations:

\[ i_d = i_{d0} + i_{d1}, \quad i_q = i_{q0} + i_{q1}, \]

(22)

\[ \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_s & \frac{L_d}{L_{mq}} \\ \frac{L_d}{L_{mq}} & \frac{R_s}{L_{mq}} \end{bmatrix} \begin{bmatrix} i_{d0} \\ i_{q0} \end{bmatrix} + \begin{bmatrix} \frac{L_d}{L_{mq}} & 0 \\ 0 & \frac{L_d}{L_{mq}} \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix}, \]

(23)

\[ \begin{bmatrix} v_{d0} \\ v_{q0} \end{bmatrix} = \begin{bmatrix} 0 & -\omega L_s \\ \omega L_s & 0 \end{bmatrix} \begin{bmatrix} i_{d0} \\ i_{q0} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \psi_a \end{bmatrix}, \]

(24)

where \( i_d \) and \( i_q \) are the currents of the \( d \)-axis and \( q \)-axis, respectively, \( v_d \) and \( v_q \) are the voltages of the \( d \)-axis,

and \( q \)-axis, respectively, \( R_s \) is the coil resistance, \( R_a \) is the equivalent iron loss resistance, \( L_d \) and \( L_q \) are the \( d \)-axis and \( q \)-axis inductances, respectively, \( \psi_a \) is the flux linkage, \( \omega_e \) is the electrical angular velocity, and \( p \) is the differential operator. In the present paper, the parameters in the equivalent circuit model were identified through bench tests. By solving the above equations, the motor torque, copper loss \( L_{Cu} \), and iron loss \( L_{Fe} \) can be obtained as follows:

\[ \tau_m = P_s \left[ \psi_a i_q + \left( L_d - L_q \right) i_{d0} i_{q0} \right], \]

(25)

\[ L_{Cu} = R_a \left( i_q^2 + i_{d0}^2 \right), \]

(26)

\[ L_{Fe} = \frac{v_{d0}^2 + v_{q0}^2}{R_e} = \frac{\omega_a^2 \left[ L_d i_{d0} i_{q0} + \left( L_d i_{d0} + \psi_a \right)^2 + \left( L_d i_{q0} \right)^2 \right]}{R_e}, \]

(27)

where \( P_s \) is the number of pairs. For ease of calculation, we treat the inverter loss \( L_{Inv} \) as a simplified loss that is proportional to the current vector \( I_{d0} \), as follows:

\[ L_{Inv} = \kappa I_{d0}. \]

(28)

The total electric loss \( L_{e} \) can be expressed as follows:

\[ L_e = L_{Cu} + L_{Fe} + L_{Inv}. \]

(29)

Figure 4 shows the electric loss at a rotational speed corresponding to 40 km/h. The simulation results
exhibit the same trend as the experimental results. Although the electric loss remains low at steady-state driving torque \( \tau_0 \), the electric loss per wheel and the mechanical resistance increase at \( \Delta L_0 \), provided that the inner and outer driving forces work in opposite directions.

### 3. Actual Vehicle Test Results

#### 3.1 Test Conditions

A passenger hybrid vehicle was customized to obtain a test vehicle equipped with four in-wheel motors. The original drive line was removed, and in-wheel motors were installed at each wheel. The major specifications are shown in Table 1.

Actual vehicle tests were conducted to validate the above formulations. For the sake of accuracy, the vehicle speed was controlled to be constant by a cruise control system and the steering angle was set to trace a constant radius circle on an asphalt road with a constant friction coefficient, \( \mu \).

First, the cornering resistance characteristics without the direct yaw moment were determined at 35 km/h. The effect of lateral acceleration was considered for circles of various turning radii.

Two experimental conditions were applied in order to consider the effect of the direct yaw moment: 20 km/h for a turning radius of 15 m (R15 m) and 40 km/h for a turning radius of 60 m (R60 m). For both conditions, the lateral acceleration was approximately 2 m/s\(^2\). The applied direct yaw moment was changed each lap, and the motor torque command values, motor rotational speeds, inverter voltage, and current were measured.

#### 3.2 Driving Resistance while Turning

The driving resistance was obtained as the total measured motor torque at each wheel, including the mechanical resistance of the reduction gears. In the present paper, we calculate the driving resistance by adding the mechanical resistance at each wheel to the driving resistance, as follows:

\[
F_w = F_x + \sum_{j=1}^{4} F_{x_j}.
\]

Figure 5 compares the experimental and simulation results for the cornering resistance without the direct yaw moment. The lateral force saturates in the high lateral acceleration region, resulting in higher cornering resistance as compared to the linear simulation result. However, the simulation model can predict the resistance precisely in the low-lateral-acceleration region.

Figure 6 shows the experimental and simulation results for R15 m and R60 m, respectively. The figure shows the simulation results for the driving resistances at the motors and tires. The difference between these resistances is the mechanical resistance. As shown in Figs. 6(a) and (b), the simulation results exhibit the same trends as the experimental results. The most

---

**Table 1** Specifications of the test vehicle.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>2,195 kg</td>
</tr>
<tr>
<td>Max. power</td>
<td>40 kW ( \times ) 4</td>
</tr>
<tr>
<td>Max. torque at axle</td>
<td>550 Nm ( \times ) 4</td>
</tr>
<tr>
<td>Max. vehicle speed</td>
<td>200 km/h</td>
</tr>
</tbody>
</table>

**Fig. 4** Electric loss characteristics.

**Fig. 5** Driving resistance in steady-state cornering.
significant point is that the driving resistance trend changes due to the cornering radius. As shown in Eq. (10), the reduction effect due to the direct yaw moment is inversely proportional to the cornering radius. Compared to the case of R60 m (Fig. 6(b)), the cornering radius of R15 m (Fig. 6(a)) is smaller and the cornering resistance decreases greatly. Although the mechanical resistance increases in the region where the inner wheels regenerate, the reduction effect at R15 m outweighs the increase in mechanical resistance. Note that a more accurate simulation can be achieved by considering the change in transmission efficiency in accordance with the operating point of the gear.

### 3.3 Power Consumption while Turning

The total power consumption and mechanical power were obtained as the product of the measured inverter current and voltage and the product of the measured motor torque and rotational speed, respectively. The total mechanical power can be calculated by adding the mechanical loss at each wheel to the mechanical power as follows:

$$P_m = \sum_{j=1}^{4} L_{mj} .$$  \hspace{1cm} (31)

The total power consumption can be calculated by adding the electric loss of each wheel to the total mechanical power as follows:

$$P_e = P_m + \sum_{j=1}^{4} L_{ej} .$$  \hspace{1cm} (32)

**Figure 7** shows the experimental and simulation results for R15 m and R60 m, respectively. The figure shows the power consumption and the mechanical power, the difference between which is the electric loss. As shown in Figs. 7(a) and (b), the simulation

![Fig. 6](image1.png)  \hspace{1cm} **Fig. 6** Driving resistance vs. direct yaw moment.

![Fig. 7](image2.png)  \hspace{1cm} **Fig. 7** Power consumption vs. direct yaw moment.
results exhibit the same trends as the experimental results. The power consumption and mechanical power exhibit similar trends between R15 m (Fig. 7(a)) and R60 m (Fig. 7(b)), regardless of the cornering radius. This is because the reduction effect of the mechanical power is not affected by the cornering radius, as shown in Eq. (17). In addition, since the test vehicle uses the same tires for the front and rear, the mechanical power is minimized without DYC. However, the mechanical power changes little due to the cornering resistance and longitudinal slip. As a result, DYC requires little additional power, provided that the inner and outer driving forces act in the same direction.

4. Considerations

4.1 Power Consumption of Typical DYCs

As an application of the simulation model, the power consumption characteristics of typical DYCs were calculated. Figures 8(a) and (b) compare the required direct yaw moment and power consumption of neutral steering control, body slip angle control \((\beta = 0)\), and load-proportional driving force control. The cornering radius was assumed to 100 m.

Neutral steering control cancels out the change in steer angle due to changes in vehicle velocity. As shown in Fig. 8(a), the required direct yaw moment increases with the vehicle velocity. As a result, the power consumption increases with the vehicle velocity, as shown in Fig. 8(b).

The body slip angle control zeroes the body slip angle. The slip angle changes from outward to inward at approximately 70 km/h, and the direct yaw moment changes from positive to negative. Except at the boundary velocity, significant power consumption is required to control the slip angle.

Load-proportional driving force control distributes the driving force in proportional to the vertical load, which spreads the work load equally between all wheels. As shown in Fig. 8(a), when the lateral acceleration increases with the vehicle velocity, the direct yaw moment increases slightly. As shown in Fig. 8(b), the power consumption increases in the higher direct yaw moment region. However, the additional power is small because load-proportional driving force control does not require regeneration of the inner wheel, even if the inner wheels lose traction. Thus, using the formulation results for the power consumption, the advantages and disadvantages of vehicle dynamics control can be calculated theoretically.

4.2 Tire Dissipation Power at the Contact Patch

This section describes the dissipation of cornering resistance and longitudinal slip. First, the cornering resistance and the other variables are re-formulated in terms of the front and rear tire slip angles \(\alpha_f\) and \(\alpha_r\), respectively, as follows:

\[
F_{yf} = -K_f\alpha_f, \quad F_{yr} = -K_r\alpha_r,
\]

\[
M_z = -2l_f F_{yf} + 2l_r F_{yr},
\]

\[
\delta_i - \delta_e = \frac{l}{R} (\alpha_e - \alpha_i),
\]
From the above formulation, the mechanical power is equal to the scalar product of the tire force vectors and tire slip velocity vectors. This is assumed to be the power loss caused by the tire slip at the tire contact patch, which is eventually dissipated as heat\(^{9-11}\) as shown in Fig. 9. Therefore, the reduction in power dissipation contributes to cornering efficiency.

5. Conclusion

In the present paper, we formulated the driving resistance and power consumption in steady-state cornering to clarify the cornering loss. Actual vehicle tests using a vehicle equipped in-wheel motors were conducted, and the obtained results validated the formulation results. The test results also clarified that the DYC can be realized with little additional power, provided that the inner and outer driving forces act in the same direction. In addition, the energy equilibrium state was explained theoretically such that the input power required to maintain steady-state cornering is dissipated at the tire contact patch.

Based on the formulation results, vehicle specifications and control methods that provide enhanced cornering efficiency will be proposed in the future.

References


(7) Yone, T. and Fujimoto, H., “Proposal of a Range Extension Control System with Arbitrary Steering

\[ \beta = \frac{l}{R} + \alpha + \delta . \]  

\[ F_{cr} = 2K, \alpha_f^2 + 2K, \alpha_s^2 - \frac{M}{R} . \]  

The first and second terms represent the local cornering resistances of the front and rear tires, respectively, and the third term represents the reduction effect by the direct yaw moment. Moreover, by substituting Eq. (37) into Eq. (17), the power of cornering resistance can be obtained as follows:

\[ P_s = -2(F_{s}, V_{sy} + F_{s}, V_{sy}) . \]  

where \( V_{sy} \) and \( V_{sx} \) are the front and rear lateral slip velocities, respectively. Representing the inner and outer longitudinal slip velocities as \( V_{sxi} \) and \( V_{sxo} \), the power of the longitudinal slip can be expressed as follows:

\[ P_{sx} = -2(F_{s}, V_{sxi} + F_{s}, V_{sxo}) . \]  

Using Eqs. (38) and (39), the mechanical power can be expressed as follows:

\[ P = \sum_{j=1}^{4} F_{j} \omega_{j} = P_{s} + P_{s} = \sum_{j=1}^{4} (F_{s}, V_{sy} + F_{s}, V_{sy}) = -\sum_{j=1}^{4} \bar{F}_{j} \bar{V}_{s} . \]
for In-wheel Motor Electric Vehicle with Four Wheel Steering”, Proc. 13th Int. Workshop Advanced Motion Control (2014), pp. 363-373


Figs. 1-4 and 6-9

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